Chapter 7
Triangles

Exercise No. 7.1

Multiple Choice Questions:

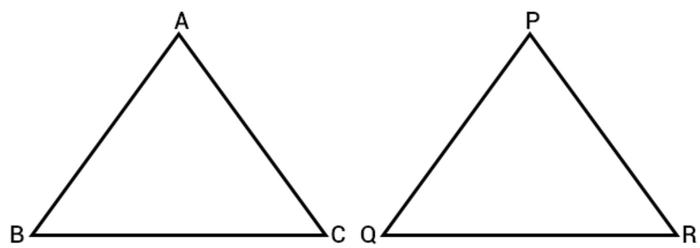
In each of the following, write the correct answer:

1. Which of the following is not a criterion for congruence of triangles?
(A) SAS
(B) ASA
(C) SSA
(D) SSS

Solution:
SSA is not a criterion for congruence of triangles.
Hence, the correct option is (C).

2. If AB = QR, BC = PR and CA = PQ, then
(A) $\Delta ABC \cong \Delta PQR$
(B) $\Delta CBA \cong \Delta PRQ$
(C) $\Delta BAC \cong \Delta RPQ$
(D) $\Delta PQR \cong \Delta BCA$

Solution:



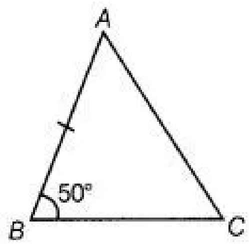
Given:
AB=QR, BC=PR and CA=PQ, then
The vertices are one-one corresponding that is P corresponding to C, Q to A and R to B, which is written as:
$P \leftrightarrow C, Q \leftrightarrow A, R \leftrightarrow B$

Under that correspondence, we have:
$\Delta CBQ \cong \Delta PRQ$
Hence, the correct option is (B).

3. In ΔABC**, AB = AC and** $\angle B = 50°$**. Then** $\angle C$ **is equal to**
(A) 40°

(B) 50°
(C) 80°
(D) 130°

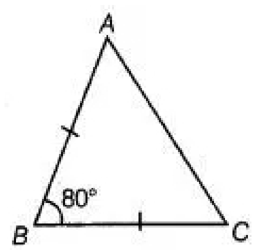
Solution:
According to the question, triangle ABC is:



AB=AC [Given]
So, $\angle C = \angle B$ [Angles opposite to equal sides are equal]
Given: $\angle B = 50°$. So, $\angle C = 50°$
Hence, the correct option is (B).

4. In ΔABC, BC = AB and $\angle B = 80°$. Then $\angle A$ is equal to
(A) 80°
(B) 40°
(C) 50°
(D) 100°

Solution:
In triangle ABC:



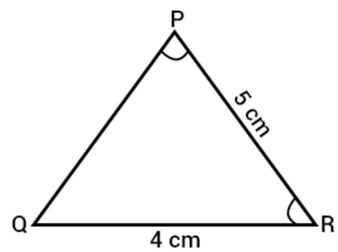
BC=AB [given]
$\angle A = \angle C$ [Since, angles opposite to equal sides are equal]
$\angle B = 80°$

Therefore, $\angle A + \angle B + \angle C = 180°$
$\angle A + 80° - \angle A = 180°$
$2\angle A = 100°$
$\angle A = \frac{100°}{2}$
$\angle A = 50°$

Hence, the correct option is (C).

5. In ΔPQR, $\angle R = \angle P$ and QR = 4 cm and PR = 5 cm. Then the length of PQ is
(A) 4 cm
(B) 5 cm
(C) 2 cm
(D) 2.5 cm

Solution:
In triangle PQR:

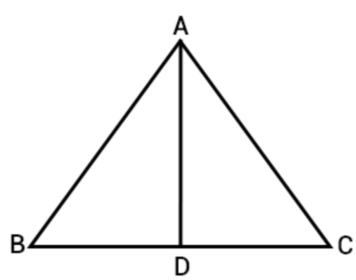


$\angle R = \angle P$ [Given]
PQ = QR [Sides opposite to equal angles are equal]

Now, QR=4cm, therefore, PQ=4cm.
Therefore, the length of the PQ is 4cm
Hence, the correct option is (A).

6. D is a point on the side BC of a ΔABC such that AD bisects $\angle BAC$. Then
(A) BD = CD
(B) BA > BD
(C) BD > BA
(D) CD > CA

Solution:
In triangle ADC,

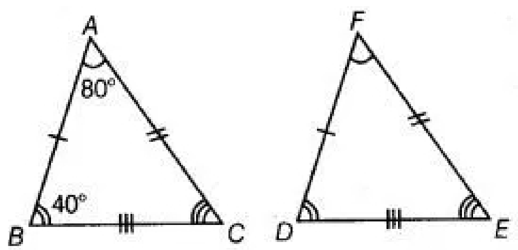


Ext.$\angle ADB >$ Int. opp $\angle DAC$
$\angle ADB > \angle BAD$ [Because: $\angle BAD = \angle DAC$]

$AB > BD$ [Side opposite to greater angle is longer]
Hence, the correct option is (B).

7. It is given that $\Delta ABC \cong \Delta FDE$ and AB = 5 cm, $\angle B = 40^\circ$ and $\angle A = 80^\circ$. Then which of the following is true?
(A) $DF = 5 \text{ cm}, \angle F = 60^\circ$
(B) $DF = 5 \text{ cm}, \angle E = 60^\circ$
(C) $DE = 5 \text{ cm}, \angle E = 60^\circ$
(D) $DE = 5 \text{ cm}, \angle D = 60^\circ$

Solution:
Given: $\Delta ABC \cong \Delta FDE$ and AB = 5 cm, $\angle B = 40^\circ$ and $\angle A = 80^\circ$



DF =AB [By CPCT]
DF =5cm
$\angle E = \angle C$ [By CPCT]
$\angle E = \angle C = 180^\circ - (\angle A + \angle B)$ [By angle sum property of a triangle ABC]
$\angle E = 180^\circ - (80^\circ + 40^\circ)$
$\angle E = 60^\circ$

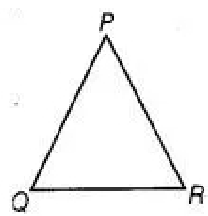
8. Two sides of a triangle are of lengths 5 cm and 1.5 cm. The length of the third side of the triangle cannot be
(A) 3.6 cm
(B) 4.1 cm
(C) 3.8 cm
(D) 3.4 cm

Solution:
Sum of any two sides of a triangle is greater than third side. So, third side of the triangle cannot be 3.4 cm because then,
1.5cm+3.4cm=4.9 cm<third side [5cm]
Hence, the correct option is (D).

9. In ΔPQR, if ∠R > ∠Q, then
(A) QR > PR

(B) PQ > PR
(C) PQ < PR
(D) QR < PR

Solution:
Given: In triangle PQR,
$\angle R > \angle Q$

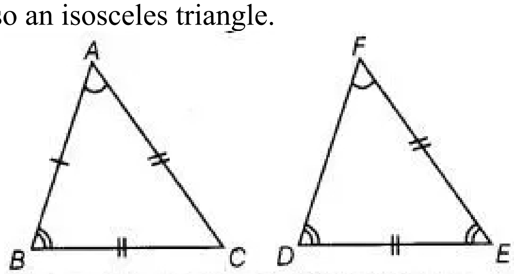


PQ>PR [side opposite to greater angle is longer]
Hence, the correct option is (B).

10. In triangles ABC and PQR, AB = AC, $\angle C = \angle P$ and $\angle B = \angle Q$. The two triangles are
(A) isosceles but not congruent
(B) isosceles and congruent
(C) congruent but not isosceles
(D) neither congruent nor isosceles

Solution:
In triangle ABC,
AB=AC [Given]
$\angle C = \angle B$ [Angles opposite to equal sides are equal]
So, in triangle ABC is an isosceles triangle.
$\angle B = \angle Q$ [Given]
$\angle C = \angle P$
$\angle P = \angle Q$ [Since, $\angle C = \angle B$]
QR=PR [Sides opposite to equal angles are equal]

So, in triangle PQR is also an isosceles triangle.



Hence, both triangle are isosceles but not congruent.
Hence, the correct option is (A).

11. In triangles ABC and DEF, AB = FD and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom if

(A) BC = EF

(B) AC = DE

(C) AC = EF

(D) BC = DE

Solution:

Given, in ΔABC and ΔDEF, AB = DF and ∠A = ∠D

As we know that, two triangles will be congruent by ASA rule, if two angles and the included side of one triangle are equal to the two angles and the included side of other triangle.

Since, AC = DE

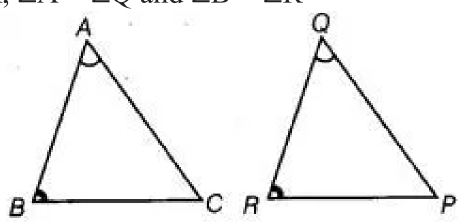
Hence, the correct option is (B).

Exercise No. 7.2

Short Answer Questions with Reasoning:

1. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle \angle B = \angle R$. Which side of ΔPQR should be equal to side AB of ΔABC so that the two triangles are congruent? Give reason for your answer.

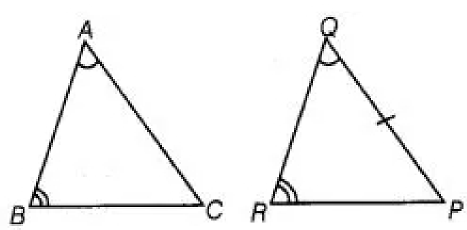
Solution:
Given: in ΔABC and ΔPQR, ∠A = ∠Q and ∠B = ∠R



Now, the triangle ABC and PQR will be congruent if AB = QR by ASA congruence rule.

2. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to side BC of ΔABC so that the two triangles are congruent? Give reason for your answer.

Solution:
Given: In triangle ABC and PQR,



$\angle A = \angle Q$ and $\angle B = \angle R$ [given]
BC = RP [For the triangle to be congruent]
Hence, it will be congruent by AAS congruence rule.

3. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Solution:
Angle must be the included angles. Hence, this statement is not true.

4. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Solution:
As we know that, the sum of any two sides of the triangle is always greater than the third side.

5. Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer.

Solution:
As we know that, the sum of any two sides of the triangle is always greater than the third side.
So,
4 cm and 3cm = 4 cm + 3 cm = 7cm that is equal to the length of third side that is 7 cm.
Therefore, this is not possible to construct a triangle with length of sides 4cm , 3 cm and 7 cm.

6. It is given that $\Delta \mathrm{ABC} \cong \Delta \mathrm{RPQ}$. Is it true to say that BC = QR? Why?

Solution:
It is false that BC = QR because BC = PQ as $\Delta ABC \cong \Delta RPQ$.

7. If $\Delta \mathrm{PQR} \cong \Delta \mathrm{EDF}$, then is it true to say that PR = EF? Give reason for your answer.

Solution:
It is true, PR=EF because this is the corresponding sides of triangle PQR and triangle EDF.

8. In $\Delta \mathrm{PQR}$, $\angle \mathrm{P} = 70°$ and $\angle \mathrm{R} = 30°$. Which side of this triangle is the longest? Give reason for your answer.

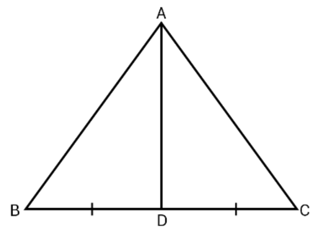
Solution:
In triangle PQR,

$$\begin{aligned}\angle Q &= 180° - (\angle P + \angle R) \\ &= 180° - (70° + 30°) \\ &= 180° - 100° \\ &= 80°\end{aligned}$$

Now, in triangle PQR, angle Q is larger and side opposite to greater angle is longer.
Therefore, PR is the longer side.

9. AD is a median of the triangle ABC. Is it true that AB + BC + CA > 2 AD? Give reason for your answer.

Solution:
In triangle ABD,
AB + BD > AD … (I)



AC+CD >AD … (II) [Sum of the lengths of any two sides of a triangle must be greater that the third side]

Adding (I) and (II), get:
AB+BD+CD+AC > 2AD
AB+BC+CA > 2AD [BD = CD as AD is median of triangle ABC]

10. M is a point on side BC of a triangle ABC such that AM is the bisector of∠∠BAC. Is it true to say that perimeter of the triangle is greater than 2 AM? Give reason for your answer.

Solution:
To prove: AB+BC+AC > 2AM

Proof: We know that sum of any two side of a triangle is greater than the third side,

Now, in triangle ABM,
AB+BM > Am…(I)

And, in triangle ACM,
AC+CM > AM …(II)

Adding (I) and (II), get:
AB+BM+AC+CM > 2AM
AB+(BM+CM) + AC > 2AM
AB + BC +AC > 2AM
Hence, it is true that the perimeter of the triangle is greater than 2AM.

11. Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give reason for your answer.

Solution:
We know that sum of any two side of a triangle is greater than the third side. So,
9 cm + 7 cm = 16 cm < 17 cm
Hence, it is not possible to construct a triangle.

12. Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer.

Solution:

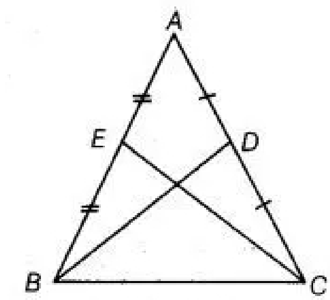
Yes, that is possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm because the sum of any two side of a triangle is greater than the third side.

Exercise No. 7.3

Short Answer Questions:

1. ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE.

Solution:
Given:
ABC is an isosceles triangle with AB = AC and BD and CE are its two medians.

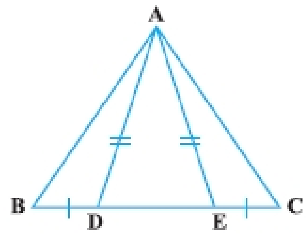


To prove: BD = CE
Proof: in triangle ABC,
AB=AC [Given]
$\frac{1}{2}AB = \frac{1}{2}AC$
AE = AD [D is the mid-point of AC and E is the mid-point of AB]

Now, in triangle ABD and triangle ACE,
AB = AC [Given]
$\angle A = \angle A$ [Common angle]
AE = AD [above proved]

Now, by SAS criterion of congruence, get:
$\Delta ABD \cong \Delta ACE$
BD = CE [CPCT]
Hence, proved.

2. In Fig., D and E are points on side BC of a ΔABC such that BD = CE and AD = AE. Show that $\Delta ABD \cong \Delta ACE$.

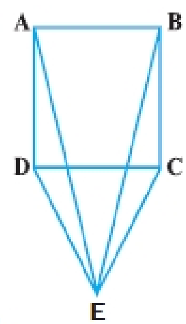


Solution:
Given in triangle ABD,
BD = CE and AD = AE
To prove that $\Delta ABD \cong \Delta ACE$

Proof:
AD = AE [Given]
$\angle ADE = \angle AED$ [Since, angle opposite to equal sides are equal] … (I)
$\angle ADB + \angle ADE = 180°$ [Linear pair axiom]
$\angle ADB = 180° - \angle ADE$
$\angle ADB = 180° - \angle AED$ [From equation (i)]

In triangle ABD and triangle ACE,
$\angle ADB = \angle AEC$ [Since, $\angle AEC + \angle AED = 180°$, linear pair axiom]
BD = CE [Given]
AD = AE [Given]
$\Delta ABD \cong \Delta ACE$ [By SAS congruence rule]

3. CDE is an equilateral triangle formed on a side CD of a square ABCD as shown in fig. Show that $\Delta ADE \cong \Delta BCE$.

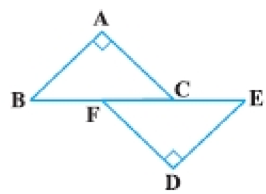


Solution:
Given in figure triangle CDE is an equilateral triangle formed on a side CD of a square ABCD.
To proof that $\Delta ADE \cong \Delta BCE$

Proof: In triangle ADE and triangle BCE,
DE = CE [Sides of an equilateral triangle]
$\angle ADE = \angle BCE$
$\angle ADC = \angle BCD = 90°$ and $\angle EDC = \angle ECD = 60°$

$\angle ADE = 90° + 60° = 150°$ and $\angle BCE = 90° + 60° = 150°$
AD = BC [Sides of a square]
$\Delta ADE \cong \Delta BCE$ [By SAS congruence rule]

4. In Fig., $BA \perp AC$**,** $DE \perp DF$ **such that BA = DE and BF = EC. Show that** $\Delta ADC \cong \Delta DEF$**.**



Solution:
See in the figure,
$BA \perp AC$, $DE \perp DF$ such that BA = DE and BF = EC

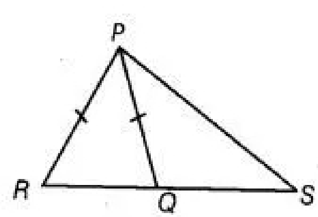
To proof that $\Delta ADC \cong \Delta DEF$
Proof:
BF = EC [Given]

Now, adding CF both sides, get:
$BF + CF = EC + CF$
BC = EF ...(I)

In triangle ABC and triangle DEF,
$\angle A = \angle D = 90°$ [$BA \perp AC$ and $DE \perp DF$]
BC = EF [from eq. (I)]
BA = DE [Given]
$\Delta ABC \cong \Delta DEF$ [By RHS congruence rule]

5. Q is a point on the side SR of a ΔPSR **such that PQ = PR. Prove that PS > PQ.**

Solution:
In triangle PSR, Q is a point on the side SR such that PQ = PR.
To proof that PS > PQ



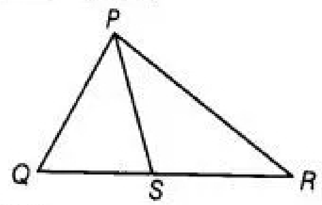
Proof: In triangle PRQ,
PQ = PR [Given]
$\angle R = \angle PQR$ …(I) [Angle opposite to equal sides are equal]

$\angle PQR > \angle S$ … (II) [Exterior angle of a triangle is greater than each of the opposite interior angle]

Now, from equation (I) and (II), get:
$\angle R > \angle S$
PS > PR [side opposite to greater angle is longer]
PS > PQ [PQ = PR]

6. S is any point on side QR of a ΔPQR. Show that: PQ + QR + RP > 2 PS.

Solution:
Given in triangle PQR, S is any point on side QR.



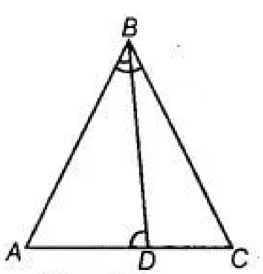
To proof that PQ + QR + RP > 2PS

Proof: In triangle PQS,
PQ + QS > PS (i) [Sum of two side of a triangle is greater than the third side]
Now, similarly in triangle PRS,
SR + RP > PS (ii) [Sum of two side of a triangle is greater than the third side]

Adding equation (I) and (II), get:
PQ + QS + SR + RP > 2PS
PQ +(QS + SR) + RP > 2PS
PQ + QR + RP > 2PS [QR = QS + SR]

7. D is any point on side AC of a ΔABC with AB = AC. Show that CD < BD.

Solution:
Given in triangle ABC, D is any point on side AC such that AB = AC.



To proof that CD < BD or BD > CD
To proof:
AC = AB [Given]

$\angle ABC = \angle ACB$ (i)[Angle opposite to equal sides are equal]

In triangle ABC and triangle DBC,

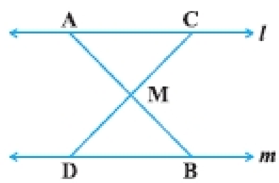
$\angle ABC > \angle DBC$ [$\angle DBC$ is a internal angle of $\angle B$]

$\angle ACB > \angle DBC$ [From equation (I)]

BD > CD [Side opposite to greater angle is longer]

CD < BD

8. In Fig., *l* || *m* and M is the mid-point of a line segment AB. Show that M is also the mid-point of any line segment CD, having its end points on *l* and *m*, respectively.



Solution:

See in the figure, *l* || *m* and M is the mid-point of a line segment AB.

To proof that MC = MD

Proof: *l*||m [Given]

$\angle BAC = \angle ABD$ [Alternate interior angles]

$\angle AMC = \angle BMD$ [Vertical opposite angle]

In triangle AMC and triangle BMD,

$\angle BAC = \angle ABD$ [Proved above]

AM = BM [Given]

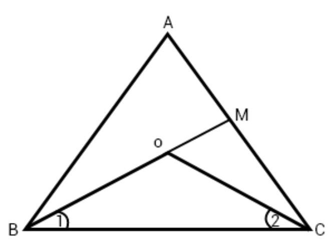
$\angle AMC = \angle BMD$ [By ASA congruence rule]

MC = MD [By CPCT]

9. Bisectors of the angles B and C of an isosceles triangle with AB = AC intersect each other at O. BO is produced to a point M. Prove that ∠MOC = ∠ABC.

Solution:

Given in the question, bisectors of the angles B and C of an isosceles triangle ABC with AB = AC intersect each other at O. Now BO is produced to a point M.

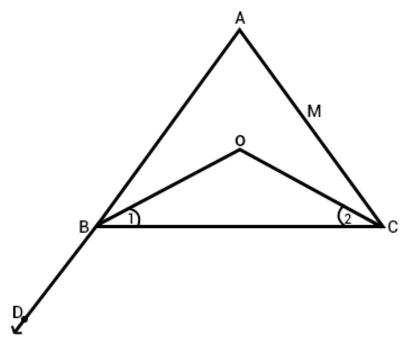


In triangle ABC,
AB = AC
$\angle ABC = \angle ACB$ [Angle opposite to equal sides of a triangle are equal]
$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

That is $\angle 1 = \angle 2$ [Since, BO and Co are bisectors of $\angle B$ and $\angle C$]
In triangle OBC, Ext. $\angle MOC = \angle 1 + \angle 2$ [Exterior angle of a triangle is equal to the sum of interior opposite angles]
Ext. $\angle MOC = 2\angle 1$ [$\angle 1 = \angle 2$]
Hence, $\angle MOC = \angle ABC$.

10. Bisectors of the angles B and C of an isosceles triangle ABC with AB = AC intersect each other at O. Show that external angle adjacent to ∠ABC is equal to ∠BOC.

Solution:
In triangle ABC,



AB = AC
So, $\angle B = \angle C$ [Angle opposite to equal sides of a triangle are equal]
$\frac{1}{2}\angle B = \frac{1}{2}\angle C$ …(I)

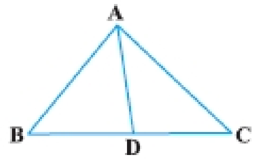
In triangle OBC,
$\angle 1 = \frac{1}{2}\angle B$
And, $\angle 2 = \frac{1}{2}\angle 2$ [By (I)]
$\angle DBC + \angle 1 + \angle OBA = 180°$ [ABD is a straight line]

In triangle OBC,
$\angle 1 + \angle 2 + \angle BOC = 180°$
$2\angle 1 + \angle BOC = 180°$ [$\angle 1 = \angle 2$]… (II)

From equation (I) and (II), get:
$\angle DBA + 2\angle 1 = 2\angle 1 + \angle BOC$

$\angle DBC = \angle BOC$

11. In Fig. 7.8, AD is the bisector of ∠BAC. Prove that AB > BD.



Solution:

In triangle ACD,

Ext. $\angle ADB > \angle DAC$ [Exterior angle of a triangle is greater than either of the interior opposite angle]

$\angle ADB > \angle BAD$

Since, in triangle ABD,

$\angle ADB > \angle BAD$

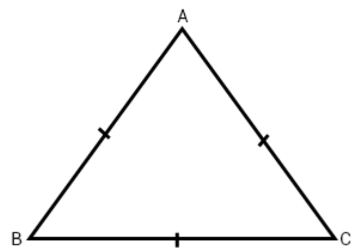
Hence, AB>BD. [In a triangle, side opposite to greater angle is longer]

Exercise No. 7.4

Long Answer Questions:

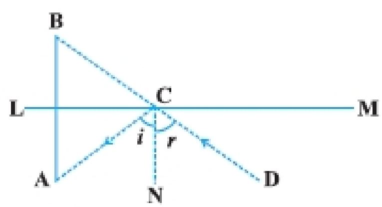
1. Find all the angles of an equilateral triangle.

Solution:
In triangle ABC,



AB = AC
$\angle C = \angle B$ … (I) [Angles opposite to equal sides of a triangle are equal]
BC = AC
$\angle A = \angle B$ … (II)
$\angle A + \angle B + \angle C = 180°$ [Angle sum property of a triangle]
$\angle A + \angle A + \angle A = 180°$ [From equation (I) and (II)]
$\angle A = \frac{180°}{3}$
$\angle A = 60°$

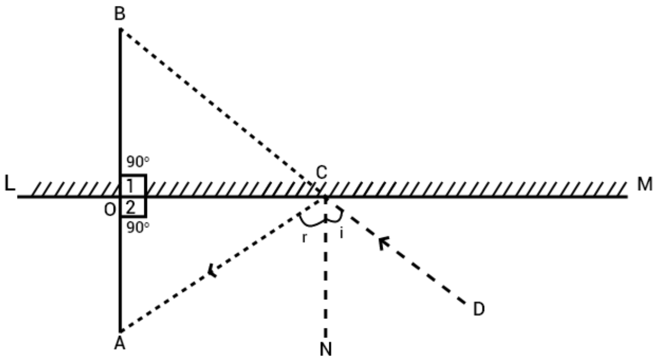
2. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in Fig. Prove that the image is as far behind the mirror as the object is in front of the mirror.
[Hint: CN is normal to the mirror. Also, angle of incidence = angle of reflection].



Solution:
Let AB intersect LM at O.
To prove: AO = BO.
Proof: $\angle i = \angle r$ …(I) [Angle of incidence = Angle of reflection]
$\angle B = \angle r$ [Corresponding angle] …(II)
Now,

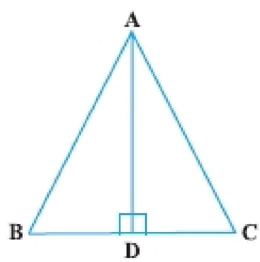
$\angle A = \angle i$ [Alternate int. $\angle s$] …(III)

Since, from equation (I), (II) and (III), get:
$\angle B = \angle A$
$\angle BCO = \angle ACO$



In triangle BOC and triangle AOC, get:
$\angle 1 = \angle 2$ [Each = $90°$]
OC = OC [Common side]
$\angle BCO = \angle ACO$ [Prove above]
$\Delta BOC \cong \Delta AOC$ [ASA congruence rule]
Hence, AO = BQ [CPCT]

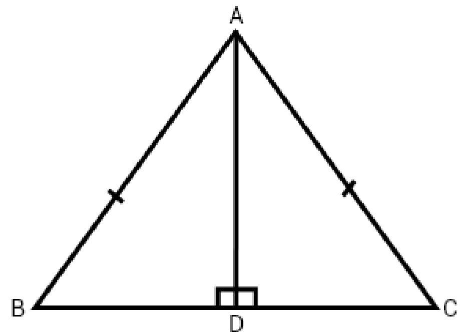
3. ABC is an isosceles triangle with AB = AC and D is a point on BC such that $AD \perp BC$ (as shown in Fig.). To prove that ∠BAD = ∠CAD, a student proceeded as follows:



In ΔABD and ΔACD,
AB = AC (Given)
$\angle B = \angle C$ (because AB = AC)
and $\angle ADB = \angle ADC$
Therefore, $\angle ABD \cong \angle ACD$ (AAS)
So, $\angle BAD = \angle CAD$ (CPCT)

What is the defect in the above arguments?
[Hint: Recall how $\angle B = \angle C$ is proved when AB = AC].

Solution:
In triangle ADB and triangle ADC, get:

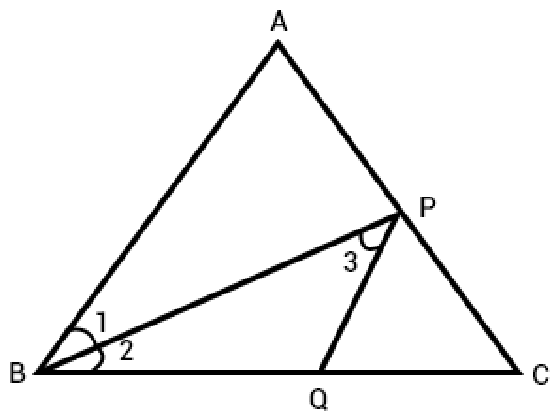


$\angle ADB = \angle ADC$ [Each equal to $90°$]
AB = AC [Given]
AD = AD [Common side]

Now, by RHS criterion of congruence, get:
$\Delta ADB \cong \Delta ADC$
So, $\angle BAD = \angle CAD$ [CPCT]
Hence, proved.

4. P is a point on the bisector of $\angle$ABC. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.

Solution:
Given in the question, P is a point on the bisector of $\angle$ABC. If the line through P, parallel to BA meet BC at Q.
To prove: BPQ is an isosceles triangle.



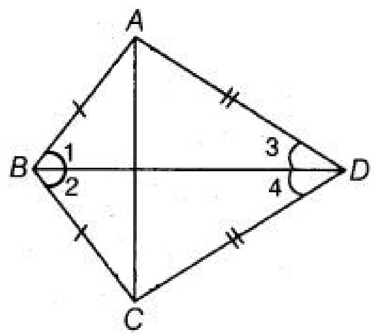
Proof: $\angle 1 = \angle 2$ …(I) [BP is the bisector of $\angle ABC$]

PQ is parallel to BA and BP cuts them. So,
$\angle 1 = \angle 3$ [Alternate interior angles as PQ||AB]
$\angle 2 = \angle 3$ [Proved above]

PQ = BQ [Sides opposite to equal angle are equal]
Hence, BPQ is an isosceles triangle.

5. ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.

Solution:
In triangle ABC and triangle CBD,

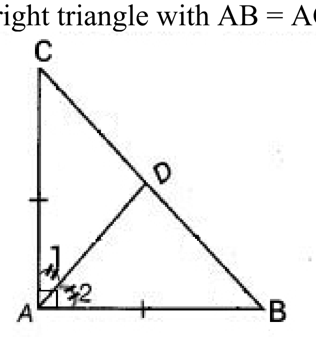


AB = BC [Given]
AD = CD [Given]
BD = BD [Common side]

So, $\Delta ABC \cong \Delta CBD$ [By SSS congruence rule]
$\angle 1 = \angle 2$ [CPCT]
And, $\angle 3 = \angle 4$
Hence, BD bisects both the angle ABC and ADC.

6. ABC is a right triangle with AB = AC. Bisector of $\angle$A meets BC at D. Prove that BC = 2 AD.

Solution:
Given in the question, ABC is a right triangle with AB = AC. Bisector of $\angle$A meets BC at D.



To prove that BC = 2AD
Proof: In right triangle ABC,
AB = AC [Given]

BC is hypotenuse. So,

$\angle BAC = 90°$

Now, in triangle CAD and triangle BAD, get:

AC = AB [Given]

$\angle 1 = \angle 2$ [AD is the bisector of $\angle A$]

AD = AD [Common side]

Now, by SAS criterion of congruence, get:

$\Delta CAD \cong \Delta BAD$

CD = BD [CPCT]

AD = BD =CD …(I) [Mid-point of hypotenuse of a rt. Triangle is equidistant from the three vertices of a triangle]

Now, BC = BD + CD

$BC = AD + AD$ [Using (I)]

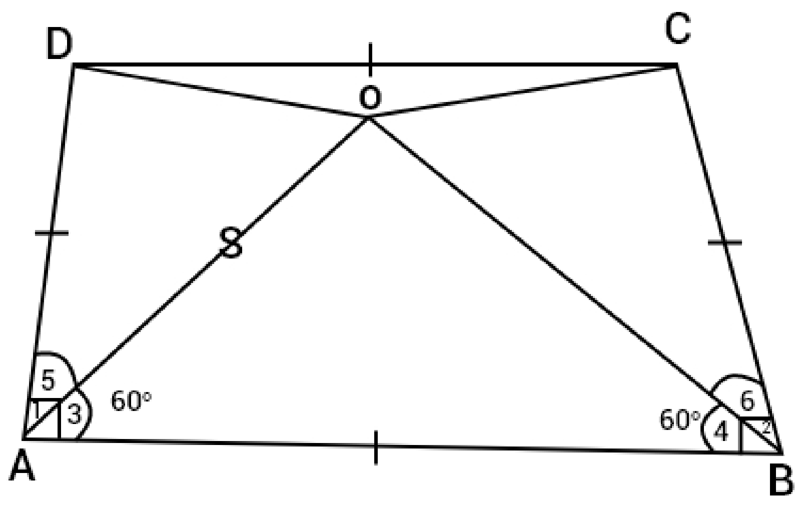
BC = 2AD

Hence, proved.

7. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that ΔOCD is an isosceles triangle.

Solution:

Given in the question, A square of ABCD and OA=OB=AB.



To prove that triangle OCD is an isosceles triangle.

Proof: In triangle ABCD,

$\angle 1 = \angle 2$ … (I) [Each equal to $90°$]

In triangle OAB,

$\angle 3 = \angle 4$ … (II) [Each equal to $60°$]

Now, subtracting equation (II) from equation (I), get:

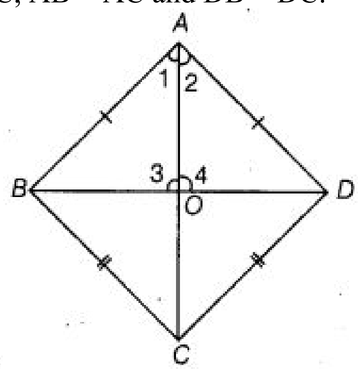
$\angle 1 - \angle 3 = \angle 2 - \angle 4$
$\angle 5 = \angle 6$

In triangle DAO and triangle CBO,
AD = BC [Given]
$\angle 5 = \angle 6$ [Proved above]
OA = OB [Given]

So, by SAS criterion of congruence, get:
$\Delta DAO \cong \Delta CBO$
OD = OC
Now, in triangle OCD is an isosceles triangle.
Hence, proved.

8. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.

Solution:
Given in the question, ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC.



To proof that AD is the perpendicular bisector of BC that is OB = OC.

Proof: In triangle BAD and triangle CAD,
AB = AC [Given]
BD = CD [Given]
AD = AD [Common side]

Now, by SSS criterion of congruence,
$\Delta BAD \cong \Delta CAD$
So, $\angle 1 = \angle 2$ [CPCT]

Now, in triangle BAO and triangle CAO,
AB = AC [Given]
$\angle 1 = \angle 2$ [Proved above]

AO = AO [Common side]

So, by SAS criterion of congruence,
$\Delta BAO \cong \Delta CAO$
Since, BO = CO [CPCT]
And, $\angle 3 = \angle 4$ [CPCT]
$\angle 3 + \angle 4 = 180°$ [Linear pair axiom]
$\angle 3 + \angle 3 = 180°$

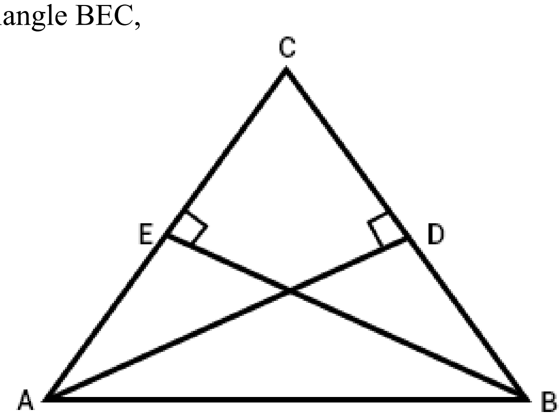
$2\angle 3 = 180°$

$\angle 3 = \frac{180°}{2}$

$\angle 3 = 90°$
Therefore, AD is perpendicular to bisector of BC.
Hence, proved.

9. ABC is an isosceles triangle in which AC = BC. AD and BE are respectively two altitudes to sides BC and AC. Prove that AE = BD.

Solution:
In triangle ADC and triangle BEC,



AC = BC [Given]…(I)
$\angle ADC = \angle BEC$ [Each is $90°$]
$\angle ACD = \angle BCE$ [Common angle]
So, $\Delta ADC \cong \angle BEC$ [By SSS congruence rule]
CE = CD …(II) [CPCT]

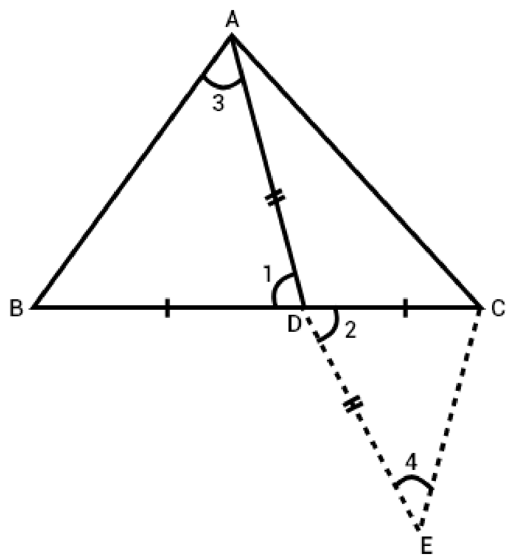
Now, Subtracting equation (II) from (I), get:
AC-AE = BC – CD
AE = BD
Hence, proved.

10. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Solution:

Given in triangle ABC with median AD,

To proof:
AB + AC > 2AD
AB + BC > 2AD
BC + AC > 2AD



Producing AD to E such that DE = AD and join EC.
Proof: In triangle ADB and triangle EDC,
AD = ED [By construction]
$\angle 1 = \angle 2$ [Vertically opposite angles are equal]
DB = DC [Given]

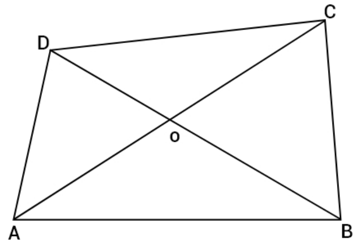
So, by SAS criterion of congruence,
$\Delta ADB \cong \Delta EDC$
AB = EC [CPCT]
And, $\angle 3 = \angle 4$ [CPCT]

Again, in triangle AEC,
AC + CE > AE [Sum of the lengths of any two sides of a triangle must be greater than the third side]
AC + CE>AD+DE
AC + CE > AD + AD [AD = DE]
AC + CE > 2AD
AC + AB > 2AD [Because AB = CE]
Hence proved.
Similarly, AB + BC > 2AD and BC + AC > 2AD.

11. Show that in a quadrilateral AB + BC + CD + DA < 2 (BD + AC)

Solution:

Given in the question, A quadrilateral,



To prove that AB + BC +CD < 2(BD+AC)

Proof: In triangle AOB,

OA + OB > AB …(I) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

In triangle BOC,

OB + OC > BC …(II) [Same reason]

In triangle COD,

OC + OD > CD…(III) [Same reason]

In triangle DOA,

OD + OA > DA…(IV) [Same reason]

Now, adding equation (I), (II), (III) and (IV), get:

OA + OB + OB + OC + OD +OD + OA > AB + BC + CD + DA

2(OA+OB+OC+OD)>AB+BC+CD+DA

2{(OA+OC)+(OB+OD)}>AB+BC+CD+DA

2(AC+BD) > AB +BC +CD +DA

AB + BC +CD +DA < 2(BD + AC)

Hence, proved.

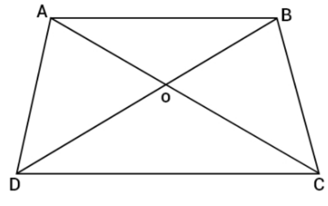
12. Show that in a quadrilateral ABCD, AB + BC + CD + DA > AC + BD

Solution:

Given in the question, a quadrilateral ABCD.

To proof that AB +BC+CD+DA>AC+BD.

Proof: In triangle ABC,



AB +BC>AC …(I) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

In triangle BCD,
BC + CD > BD …(II) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

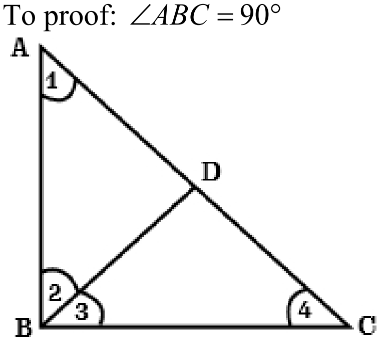
In triangle CDA,
AD+DA>AC …(III) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

Similarly, in triangle DAB,
AD + AB >BD …(IV) [Sum of the lengths of any two sides of a triangle must be greater than the third side]

Now, adding equation (I), (II), (III) and (IV), get:
AB + BC +BC +CD+CD+DA+AD+AB > AC +BD +AC +BD
2AB +2BC +2CD > 2AC +2BD
2(AB + BC +CD +DA) > 2(AC +BD)
AB +BC +CD +DA > AC +BD
Hence, proved.

13. In a triangle ABC, D is the mid-point of side AC such that $BD = \frac{1}{2}AC$. Show that ∠ABC is a right angle.

Solution:
Given: D is the mid-point of side AC.
To proof: $\angle ABC = 90°$



Proof: AD = DC
And, BD $= \frac{1}{2}AC = AD$ [D is the mid-point of side AC]
BD = AD = DC

In triangle ABD,
BD = AD
$\angle 1 = \angle 2$ … (I) [Angles opposite to equal sides are equal]