Chapter 12

Heron's Formula

Exercise No. 12.1

Multiple Choice Questions:

1. An isosceles right triangle has area 8 cm^2. The length of its hypotenuse is

(A) $\sqrt{32}$ cm

(B) $\sqrt{16}$ cm

(C) $\sqrt{48}$ cm

(D) $\sqrt{24}$ cm

Solution:

Given: An isosceles right triangle has area 8 cm^2.

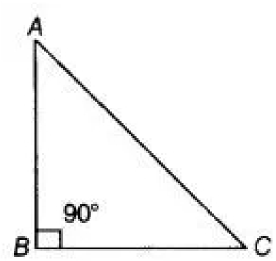
Area of an isosceles right triangle $=\frac{1}{2}\times\text{Base}\times\text{Height}$

So, $8=\frac{1}{2}\times\text{Base}\times\text{Height}$

$(\text{Base})^2=16$ [Base=height, as triangle is an isosceles]

Base $=\sqrt{16}$

Base =4cm



See the triangle ABC, using Pythagoras theorem:

$$AC^2+AB^2+BC^2=4^2+4^2$$
$$=16+16$$
$$AC^2=32$$
$$AC=\sqrt{32}$$

Therefore, the length of its hypotenuse is $\sqrt{32}$.

Hence, the correct option is (A).

2. The perimeter of an equilateral triangle is 60 m. The area is

(A) $10\sqrt{3}\text{ m}^2$

(B) $15\sqrt{3}\text{ m}^2$

(C) $20\sqrt{3}\text{ m}^2$

(D) $100\sqrt{3}\text{ m}^2$

Solution:

Given: The perimeter of an equilateral triangle is 60 m.

Suppose that each side of an equilateral be a.

$a + a + a = 60\text{m}$

$3a = 60\text{m}$

$a = 20\text{m}$

Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{Side})^2$

$= \frac{\sqrt{3}}{4} \times (20)^2$

$= 100\sqrt{3}\text{m}^2$

Therefore, the area of the triangle is $100\sqrt{3}\text{m}^2$.

3. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is

(A) 1322 cm^2

(B) 1311 cm^2

(C) 1344 cm^2

(D) 1392 cm^2

Solution:

The sides of a triangle are a = 56cm, b=60cm and c=52cm.

So, semi-perimeter of a triangle will be:

$s = \frac{a+b+c}{2}$

$= \frac{56+60+52}{2}$

$= \frac{168}{2}$

$= 84cm$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ [By heron's formula]

$= \sqrt{84(84-56)(84-60)(84-52)}$

$= \sqrt{84 \times 28 \times 24 \times 32}$

$= \sqrt{4 \times 7 \times 3 \times 4 \times 7 \times 4 \times 2 \times 3 \times 4 \times 4 \times 2}$

$= \sqrt{4^6 \times 7^2 \times 3^2}$

$= 4^3 \times 7 \times 3$

$= 1344\text{cm}^2$

Hence, the correct option is (C).

4. The area of an equilateral triangle with side $2\sqrt{3}$ cm is
(A) 5.196 cm^2
(B) 0.866 cm^2
(C) 3.496 cm^2
(D) 1.732 cm^2

Solution:
Given: The side of an equilateral triangle is $2\sqrt{3}$ cm.

Now, area of an equilateral triangle$=\frac{\sqrt{3}}{4}(\text{Side})^2$

$$=\frac{\sqrt{3}}{4}\times\left(2\sqrt{3}\right)^2$$
$$=\frac{\sqrt{3}}{4}\times4\times3$$
$$=3\sqrt{3}$$
$$=3\times1.732$$
$$=5.196\text{cm}^2$$

Hence, the area of an equilateral triangle is 5.196cm^2.
Hence, the correct option is (A).

5. The length of each side of an equilateral triangle having an area of $9\sqrt{3}$ cm^2 is
(A) 8 cm
(B) 36 cm
(C) 4 cm
(D) 6 cm

Solution:
Given: area of an equilateral triangle = $9\sqrt{3}\text{cm}^2$

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}\times(\text{Side})^2$

$$\frac{\sqrt{3}}{4}\times(\text{Side})^2=9\sqrt{3}$$
$$(\text{Side})^2=9\times4$$
$$\text{Side}=\sqrt{9\times4}$$
$$\text{Side}=3\times2$$
$$\text{Side}=6\text{cm}$$

Therefore, the length of an equilateral triangle is 6 cm.
Hence, the correct option is (D).

6. If the area of an equilateral triangle is $16\sqrt{3}\text{ cm}^2$, then the perimeter of the triangle is

(A) 48 cm

(B) 24 cm

(C) 12 cm

(D) 306 cm

Solution:

Given: The area of an equilateral triangle is $16\sqrt{3}\text{ cm}^2$.

Area of equilateral triangle $=\frac{\sqrt{3}}{4}\times(\text{side})^2$

$$16\sqrt{3}=\frac{\sqrt{3}}{4}\times(\text{side})^2$$

$$(\text{Side})^2=\frac{16\sqrt{3}\times 4}{\sqrt{3}}$$

$$=64$$

$$\text{Side}=\sqrt{64}$$

$$\text{Side}=8\text{cm}$$

Therefore, the perimeter of triangle 8 + 8 + 8 = 24cm

Hence, the correct option is (B).

7. The sides of a triangle are 35 cm, 54 cm and 61 cm, respectively. The length of its longest altitude

(A) $16\sqrt{5}\text{ cm}$

(B) $10\sqrt{5}\text{ cm}$

(C) $24\sqrt{5}\text{ cm}$

(D) 28 cm

Solution:

Given: The sides of a triangle are a= 35 cm, b=54 cm and c=61 cm, respectively. So, semi-perimeter of a triangle is:

$$s=\frac{a+b+c}{2}=\frac{35+54+61}{2}=\frac{150}{2}=75$$

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{75(75-35)(75-54)(75-61)}$$

$$= \sqrt{75 \times 40 \times 21 \times 14}$$

$$= \sqrt{5 \times 5 \times 3 \times 2 \times 2 \times 2 \times 5 \times 3 \times 7 \times 7 \times 2}$$

$$= 5 \times 3 \times 2 \times 2 \times 7\sqrt{5}$$

$$= 420\sqrt{5}$$

As know that,

Area of triangle ABC= $\frac{1}{2} \times \text{Base} \times \text{Altitude}$

$$\frac{1}{2} \times 35 \times \text{Altitude} = 420\sqrt{5}$$

$$\text{Altitude} = \frac{420\sqrt{5} \times 2}{35}$$

$$\text{Altitude} = 24\sqrt{5}$$

Therefore, the length of altitude is $24\sqrt{5}$.
Hence, the correct option is (C).

8. The area of an isosceles triangle having base 2 cm and the length of one of the equal sides 4 cm, is

(A) $\sqrt{15}\ \text{cm}^2$

(B) $\sqrt{\frac{15}{2}}\ \text{cm}^2$

(C) $2\sqrt{15}\ \text{cm}^2$

(D) $4\sqrt{15}\ \text{cm}^2$

Solution:

Given: The length of side be a = 2cm and b = 4cm.
As we know that,

Area of an isosceles triangle = $\frac{a}{4}\sqrt{4b^2 - a^2}$

$$= \frac{2\sqrt{4 \times (4)^2 - 2^2}}{4}$$

$$= \frac{\sqrt{64-4}}{2}$$

$$= \frac{\sqrt{60}}{2}$$
$$= \frac{2\sqrt{15}}{2}$$
$$= \sqrt{15}\text{cm}^2$$

Hence, the correct option is (A).

9. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 9 paise per cm² is
(A) Rs 2.00
(B) Rs 2.16
(C) Rs 2.48
(D) Rs 3.00

Solution:
Given: The edges of a triangular board are a=6 cm, b=8 cm and c=10 cm.
Now, semi-perimeter of a triangular board will be:

$$s = \frac{a+b+c}{2}$$
$$= \frac{6+8+10}{2}$$
$$= \frac{24}{2}$$
$$= 12cm$$

Now, by Heron's formula:

Area of a triangle board = $\sqrt{s(s-a)(s-b)(s-c)}$
$$= \sqrt{12(12-6)(12-8)(12-10)}$$
$$= \sqrt{12\times 6\times 4\times 2}$$
$$= \sqrt{12^2 \times 2^2}$$
$$= 12\times 2$$
$$= 24\text{cm}^2$$

As, the cost of painting for area $1\,\text{cm}^2$ = Rs. 0.09
So, Cost of paint for area 24 $\text{cm}^2 = 0.09\times 24$ = Rs. 2.16
Therefore, the cost of a triangular board is Rs. 2.16.
Hence, the correct option is (B).

Exercise No. 12.2

Short Answer Questions with Reasoning:

Write True or False and justify your answer:

1. The area of a triangle with base 4 cm and height 6 cm is 24 cm^2.

Solution:
Given: The base and height of a triangle are 4 cm and 6 cm respectively.

As we know that, area of a triangle = $\frac{1}{2}\times \text{Base}\times \text{Height}$

$$=\frac{1}{2}\times 4\times 6$$

$$=12\text{cm}^2$$

Hence, the given statement is false.

2. The area of ΔABC is 8 cm^2 in which AB = AC = 4 cm and $\angle A = 90°$.

Solution:
Area of a triangle = $\frac{1}{2}\times \text{Base}\times \text{Height}$

$$=\frac{1}{2}\times 4\times 4$$

$$=8\text{cm}^2$$

Hence, the given statement is true.

3. The area of the isosceles triangle is $\frac{5}{4}\sqrt{11}$ cm^2, if the perimeter is 11 cm and the base is 5 cm.

Solution:
Suppose that side of isosceles triangle be a.
Now, perimeter of an isosceles triangle:
2s = 5 + a + a [2s = a + b + c]
11= 5+2a
2a = 11 – 5
2a=6
a=3

Now, the formula of an area of isosceles triangle = $\frac{a}{4}\sqrt{4b^2-a^2}$

So, area of an isosceles triangle = $\dfrac{5\sqrt{4\times(3)^2-(5)^2}}{4}$

$$=\frac{5\sqrt{4\times 9-25}}{4}$$

$$=5\times\frac{\sqrt{36-25}}{4}$$

$$=\frac{5\sqrt{11}}{4}\text{cm}^2$$

Hence, the given statement is true.

4. The area of the equilateral triangle is $20\sqrt{3}\ \text{cm}^2$ whose each side is 8 cm.

Solution:

Given, side of an equilateral triangle be 8 cm.

Area of the equilateral triangle = $\dfrac{\sqrt{3}}{4}$ (Side)^2

$$=\frac{\sqrt{3}}{4}\times (8)^2$$

$$=\frac{64}{3}\sqrt{3}\ [\because \text{side} = 8 \text{ cm}]$$

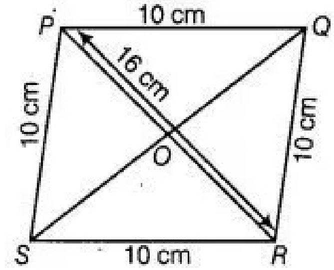
$$= 16\sqrt{3}\ \text{cm}^2$$

Hence, the given statement is false.

5. If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is $96\ \text{cm}^2$.

Solution:

Let PQRS be the rhombus whose one diagonal is 16 cm, the area of the rhombus is 10 cm.



As we know that diagonal of a rhombus bisect each other at right angles. So,
OA = OC = 8cm and OB = OD

Now, in triangle AOB, $\angle AOB = 90°$

So, $AB^2 = OA^2 + OB^2$ [By Pythagoras theorem]

$AB^2 = OA^2 + OB^2$

$OB^2 = AB^2 - OA^2$

$= (10)^2 - 8^2$

$= 100 - 64$

$= 36$

So, $OB = \sqrt{36} = 6$

Also,

$OB = 2(OA) = 2 \times 6$

$= 12cm$

Therefore, area of rhombus = $\frac{1}{2} \times$ Products of diagonals

$= \frac{1}{2} \times 16 \times 12$

$= 96\text{cm}^2$

Hence, the given statement is true.

6. The base and the corresponding altitude of a parallelogram are 10 cm and 3.5 cm, respectively. The area of the parallelogram is 30 cm^2.

Solution:

Given, parallelogram in which base = 10 cm and altitude = 3.5 cm

Area of a parallelogram = Base x Altitude

= 10 x 3.5

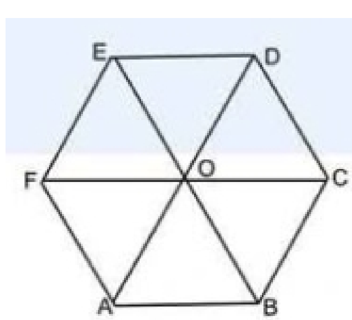
= 35 cm^2

Hence, the given statement is false.

7. The area of a regular hexagon of side '*a*' is the sum of the areas of the five equilateral triangles with side *a*.

Solution:

Given: The side of a regular hexagon is 'a'.



As we know that the regular hexagon is divided into six equilateral triangles. So,

Area of regular hexagon = Sum of area of the six equilateral triangles.
Hence, the given statement is false.

8. The cost of levelling the ground in the form of a triangle having the sides 51 m, 37 m and 20 m at the rate of Rs 3 per m^2 is Rs 918.

Solution:
Given: The sides of the ground are a = 51m, b = 37cm, and c = 20cm. Now, the semi-parameter(s) of ground is:

$2s = a + b + c$

$2s = 51\text{m}+37\text{m}+20\text{m}$

$2\text{s} = 108\text{m}$

$\text{s} = \frac{108\text{m}}{2}$

$s = 54\text{m}$

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-51)(54-37)(54-20)}$

$= \sqrt{54\times3\times17\times34}$

$= \sqrt{3\times3\times3\times2\times3\times17\times17\times2}$

$= 3\times3\times17\times2$

$= 306\text{m}^2$

The cost of levelling of 1 m^2 area is Rs. 3.
So, cost of levelling the ground of 306 m^2 area = Rs. 3×306= Rs. 918
Hence, the given statement is true.

9. In a triangle, the sides are given as 11 cm, 12 cm and 13 cm. The length of the altitude is 10.25 cm corresponding to the side having length 12 cm.

Solution:
Given: The length of the altitude is 10.25. And in a triangle, the sides are a=11cm, b=12cm and c = 13cm.
So, semi-perimeter(s) will be:

$2s = a + b + c$

$2s = 11\text{cm} + 12\text{cm} + 13\text{cm}$

$2s = 36\text{cm}$

$s = \frac{36}{2}$

$s = 18\text{cm}$

So, area of triangle $= \frac{2 \times \text{Area of } \Delta}{\text{Base}}$

$$= \frac{2 \times 6\sqrt{105}}{12}$$

$$= \sqrt{105}$$

$$= 10.25$$

Hence, the given statement is true.

Exercise No. 12.3

Short Answer Questions:

1 Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m².

Solution:

Given: The sides of the ground are a = 50m, b = 65m, and c = 65m. Now, the semi-parameter(s) of the cost of levelling is:

$2s = a + b + c$

$2s = 50\text{m}+65\text{m}+65\text{m}$

$2\text{s} = 180\text{m}$

$\text{s} = \dfrac{180\text{m}}{2}$

$s = 90\text{m}$

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{90(90-50)(90-65)(90-65)}$

$= \sqrt{90 \times 40 \times 25 \times 25}$

$= 3 \times 2 \times 10 \times 25$

$= 6 \times 250$

$= 1500\text{m}^2$

The cost of laying grass 1 m^2 area is Rs. 7.

Therefore, the cost of levelling grass per 1500m^2 = Rs. 7×1500= Rs. 10500

2 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs 2000 per m² a year. A company hired one of its walls for 6 months. How much rent did it pay?

Solution:

Let the sides of a triangular walls are a = 13m, b = 14m and c = 15m.

Now, the semi-perimeter of triangular side wall,

$s = \dfrac{a+b+c}{2}$

$= \dfrac{13+14+15}{2}$

$= 21\text{m}$

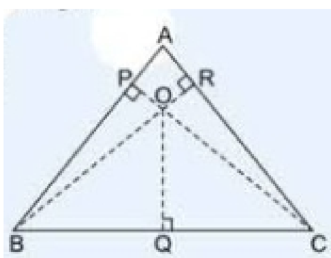
Now, area of triangular wall $= \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$$= \sqrt{21(21-13)(21-14)(21-15)}$$
$$= \sqrt{21\times(21-13)\times(21-14)\times(21-15)}$$
$$= \sqrt{21\times8\times7\times6}$$
$$= \sqrt{21\times4\times2\times7\times3\times2}$$
$$= \sqrt{21^2\times4^2}$$
$$= 21\times4$$
$$= 84\text{m}^2$$

The advertisement yield earning per year for 1 m^2 area is Rs. 2000.
Therefore, advertisement yield earning per year on 84 $m^2 = 2000\times84$= Rs. 168000.
According to the question, the company hired one of its walls for 6 months, therefor company pay the rent $= \frac{1}{2}\times168000 = \text{Rs. } 84000$.
Hence, the company paid rent $\text{Rs. } 84000$.

3 From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Solution:
Let ABC be an equilateral triangle, O be the interior point and OP=14cm, OQ = 10cm and OR = 6cm. Also, sides of an equilateral triangle be *a* m.



Area of triangle OAB = $\frac{1}{2}\times AB\times OP$ [Area of a triangle $= \frac{1}{2}\times(\text{Base}\times\text{Height})$]
$$= \frac{1}{2}\times a\times14$$
$$= 7a\text{cm}^2$$

Similarly, Area of triangle OBC = $\frac{1}{2}\times BC\times OQ$
$$= \frac{1}{2}\times a\times10$$
$$= 5a\text{cm}^2$$

Again, area of triangle OAC $=\frac{1}{2}\times AC\times OR$

$$=\frac{1}{2}\times a\times 6$$

$$=3a\text{cm}^2$$

See the given figure, area of equilateral triangle ABC = Area of $\left(\Delta OAB+\Delta OBC+\Delta OAC\right)$

$$=\left(7a+5a+3a\right)\text{cm}^2$$

$$=15a\text{cm}^2$$

Now, semi-perimeter of triangle ABC is:

$$s=\frac{a+a+a}{2}$$

$$s=\frac{3a}{2}cm$$

As, area of equilateral triangle ABC $=\sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)}$ [By Heron's formula]

$$=\sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$=\sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}}$$

$$=\frac{\sqrt{3}}{4}a^2\ \ldots\text{(II)}$$

According to the equation (I) and (II), get:

$$\frac{\sqrt{3}}{4}a^2=15a$$

$$a=\frac{15\times 4}{\sqrt{3}}$$

$$a=\frac{60}{\sqrt{3}}$$

$$a=20\sqrt{3}$$

Putting $a=20\sqrt{3}$ in equation (II), get:

Area of triangle ABC $=\frac{\sqrt{3}}{4}\times\left(20\sqrt{3}\right)^2$

$$=\frac{\sqrt{3}}{4}\times 400\times 3$$

$$=300\sqrt{3}cm^2$$

Hence, the area of an equilateral triangle is $300\sqrt{3}\text{cm}^2$.

4 The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3 : 2. Find the area of the triangle.

Solution:

Given: Perimeter of triangle= 32cm

The ratio of the equal side to its base of an isosceles triangle is 3 : 2. Let sides of an isosceles triangle be 3x, 3x and 2x.

So, perimeter of the triangle = $3x + 3x + 2x = 8x$

32 = 8x

$x = \frac{32}{8}$

$x = 4$

Since, the sides of the isosceles triangle are $3\times4=12$, $3\times4=12$ and $2\times4=8cm$.

Now, semi-perimeter of triangle will be:

$$s = \frac{12+12+8}{2}$$
$$= \frac{32}{2}$$
$$= 16cm$$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

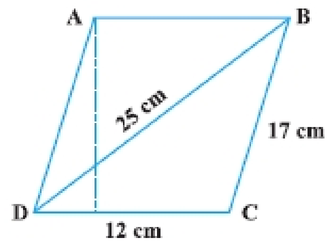
$$= \sqrt{16(16-12)(16-12)(16-8)}$$
$$= \sqrt{16\times4\times4\times8}$$
$$= 4\times4\times2\sqrt{2}cm^2$$
$$= 32\sqrt{2}$$

Therefore, the area of an isosceles triangle ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16(16-12)(16-12)(16-8)}$$
$$= \sqrt{16\times4\times4\times8}$$
$$= 32\sqrt{2}cm^2$$

Therefore, the area of an isosceles triangle is $32\sqrt{2}\text{cm}^2$.

5 Find the area of a parallelogram given in Fig. Also find the length of the altitude from vertex A on the side DC.



Solution:

Let the sides of a triangle BCD are a = 12cm, b = 17 cm and c = 25 cm and altitude of a parallelogram is h.

Area of parallelogram, ABCD = 2 (Area of triangle BCD) …(I)

Now, semi-perimeter(s) of triangle BCD will be:

$$s=\frac{a+b+c}{2}$$
$$=\frac{12+17+25}{2}$$
$$=\frac{54}{2}$$
$$=27cm$$

Area of triangle BCD $=\sqrt{s(s-a)(s-b)(s-c)}$ [By heron's formula]

$$=\sqrt{27(27-12)(27-17)(27-25)}$$
$$=\sqrt{27\times15\times10\times2}$$
$$=\sqrt{9\times3\times3\times5\times5\times2\times2}$$
$$=3\times3\times5\times2\text{cm}^2$$
$$=90\text{cm}^2$$

So, area of parallelogram ABCD = $2\times$Area of triangle BCD

$=2\times90\text{cm}^2$

$=180\text{cm}^2$ …(II)

As, Area of parallelogram ABCD = Base $\times$ Altitude

$180 = \text{DC}\times\text{h}$

$180 = 12\times\text{h}$

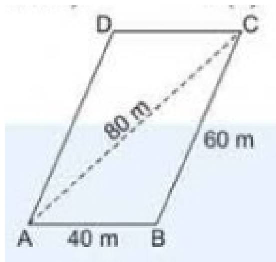
$$h=\frac{180}{12}$$
$$h=15\text{cm}$$

Therefore, the area of parallelogram is $180\,\text{cm}^2$ and the length of altitude is 15 cm.

6 A field in the form of a parallelogram has sides 60 m and 40 m and one of its diagonals is 80 m long. Find the area of the parallelogram.

Solution:

Given: Let a field in the form of a parallelogram ABCD has sides 60 m and 40 m and one of its diagonals is 80 m long.



See the figure, in triangle ABC, let a = 40m, b = 60 m and c = 80m.

Now, semi perimeter(s) of triangle ABC:

$$s = \frac{a+b+c}{2}$$
$$= \frac{40+60+80}{2}$$
$$= \frac{180}{2}$$
$$= 90\text{m}$$

So, area of triangle ABC will be $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{90(90-40)(90-60)(90-80)}$$
$$= \sqrt{90 \times 50 \times 30 \times 10}$$
$$= \sqrt{3 \times 30 \times 5 \times 10 \times 30 \times 10}$$
$$= 300\sqrt{15}$$
$$= 1161.895m^2$$

Now, from equation (I),

Area of parallelogram ABCD = $2 \times 1161.895\text{m}^2 = 2323.79m^2$.

Therefore, the area of parallelogram ABCD is $2323.79m^2$.

7 The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8. Find the area of the triangular field.

Solution:

Given: The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8.

According to the question, Let the sides in meters are a= 6x, b= 7x and c=8x.

So, perimeter of the triangle=6x+7x+8x

420 = 21x

$x = \frac{420}{21}$

$x = 20$

Since, the sides of the triangular field are $a = 6 \times 20\text{cm} = 120\text{m}$, $b = 7 \times 20m = 140m$ and $c = 8 \times 20m = 160m$.

Now, semi-perimeter(s) of triangle will be:

$s = \frac{1}{2} \times 420m$

$= 210m$

Area of the triangle field = $\sqrt{s(s-a)(s-b)(s-c)}$ [Using Heron's formula]

$= \sqrt{210(210-120)(210-140)(210-160)}$

$= \sqrt{210 \times 90 \times 70 \times 50}$

$= 100\sqrt{7 \times 3 \times 3^2 \times 7 \times 5}$

$= 100 \times 7 \times 3 \times \sqrt{15}$

$= 2100\sqrt{15}$

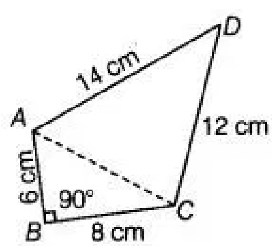
Therefore, the area of the triangular field is $2100\sqrt{15}$.

8 The sides of a quadrilateral ABCD are 6 cm, 8 cm, 12 cm and 14 cm (taken in order) respectively, and the angle between the first two sides is a right angle. Find its area.

Solution:

Given: The sides of a quadrilateral ABCD are AB =6 cm, BC = 8 cm, and CD= 12 cm and DA= 14 cm.

Construction: Join AC.



In the right triangle ABC, whose angle B is right angle. So,

$AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$AC^2 = 6^2 + 8^2$

$AC^2 = 36 + 64$

$AC = \sqrt{100}$

$AC = 10$

Area of quadrilateral ABCD = Area of triangle ABC + Area of triangle ACD

Now, area of triangle ABC= $\frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 6 \times 8$

$= 24\text{cm}^2$

In triangle ACD, let AC = a = 10 cm, CD = b= 12 cm, and DA =c=14cm.

Now, semi-perimeter of triangle ACD will be:

$s = \frac{a+b+c}{2}$

$= \frac{10+12+14}{2}$

$= \frac{36}{2}$

$= 18cm$

So, area of triangle ACD = $\sqrt{s(s-a)(s-b)(s-c)}$ [By heron's formula]

$= \sqrt{18(18-10)(18-12)(18-14)}$

$= \sqrt{18 \times 8 \times 6 \times 4}$

$= \sqrt{(3)^2 \times 2 \times 4 \times 2 \times 3 \times 2 \times 4}$

$= 3 \times 4 \times 2\sqrt{3 \times 2}$

$= 24\sqrt{6}\text{cm}^2$

Hence, the area of the quadrilateral ABCD is $24\sqrt{6}\text{cm}^2$.

9 A rhombus shaped sheet with perimeter 40 cm and one diagonal 12 cm, is painted on both sides at the rate of Rs 5 per m^2. Find the cost of painting.

Solution:

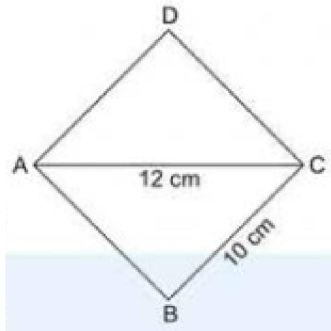
Given: One diagonal = 12 cm, Perimeter of rhombus = 40 cm

So,

$4 \times \text{Side} = 40$

$\text{side} = \frac{40}{4}$

$\text{Side} = 10\text{cm}$



In triangle ABC, let a = 10 cm, b= 10 cm, and c = 12 cm.
As we know that rhombus is also a parallelogram, so its diagonal divide it into two congruent triangles of equal area. So,
Area of rhombus = 2 (Area of triangle ABC)

Now, Semi-perimeter of triangle ABC will be:

$$s = \frac{a+b+c}{2}$$
$$= \frac{10+10+12}{2}$$
$$= \frac{32}{2}$$
$$= 16\text{cm}$$

So, area of triangle ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

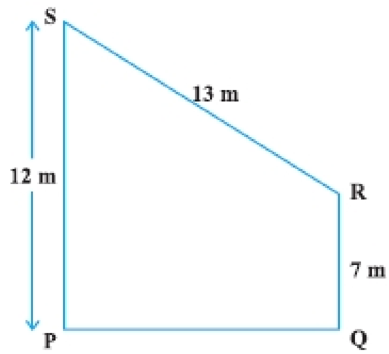
$$= \sqrt{16(16-10)(16-10)(16-12)}$$
$$= \sqrt{16 \times 6 \times 6 \times 4}$$
$$= \sqrt{2304}$$
$$= 48cm^2$$

Since, area of rhombus = 2 (Area of triangle ABC)

$$= 2 \times 48\text{cm}^2$$
$$= 96\text{cm}^2$$

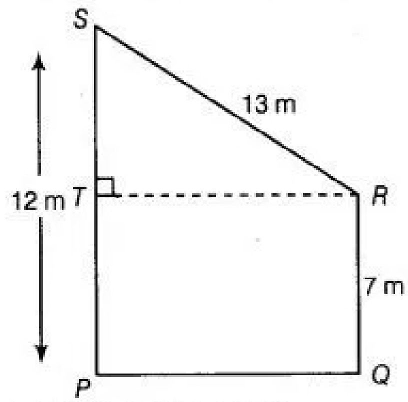
The cost of painting of the sheet is Rs. 5 per m^2.
Therefore, cost of painting both sides of rhombus shaped sheet ABCD = $\text{Rs.}(2 \times 5 \times 96) = \text{Rs. } 960$.

10 Find the area of the trapezium PQRS with height PQ given in Fig.



Solution:

Let PQRS is a trapezium, in which draw a line RT perpendicular to PS.



See the figure, ST = PS – TP = 12 – 7 = 5cm

So, in right triangle STR,

$(SR)^2 = (ST)^2 + (TR)^2$ [By using Pythagoras theorem]

$(13)^2 = (5)^2 + (TR)^2$

$(TR)^2 = 169 - 25$

$(TR)^2 = 144$

$TR = 12m$

Now, area of triangle STR $= \frac{1}{2} \times TR \times TS$ [area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 12 \times 5$

$= 30m^2$

As, area of rectangle PQRT = $PQ \times RQ = 12 \times 7 = 84m^2$

Now, area of trapezium = Area of DSTR + Area of rectangle PQRT

$= 30 + 84$

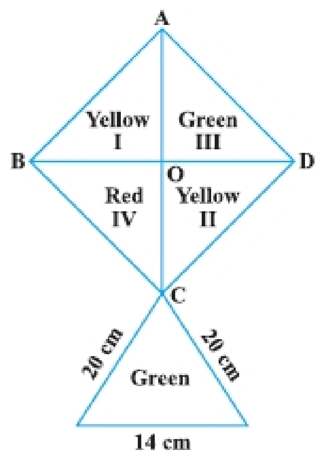
$= 114m^2$

Therefore, the area of trapezium is $114m^2$.

Exercise No. 12.4

Long Answer Questions:

1. How much paper of each shade is needed to make a kite given in Fig., in which ABCD is a square with diagonal 44 cm?



Solution:
Given: Diagonal of square ABCD = 44 cm
Also, ABCD is a square. So, AB = BC = CD = DA

Now, in triangle ACD,

$$AC^2 = AD^2 + DC^2$$
$$44^2 = AD^2 + AD^2$$
$$2AD^2 = 44 \times 44$$
$$2AD^2 = 22 \times 2 \times 22 \times 2$$
$$AD^2 = 22 \times 2 \times 22$$
$$AD = \sqrt{22 \times 2 \times 22}$$
$$AD = 22\sqrt{2}$$

Now, area of square ABCD = Side $\times$ Side $= 22\sqrt{2} \times 22\sqrt{2} = 968\text{cm}^2$
Since, area of square is divided into four parts.
Now, the area of paper of Red shade needed to make the kite is: $= \frac{1}{4} \times 968cm^2 = 242cm^2$
Also, area of green portion is:
$$= \frac{1}{4} \times 968cm^2$$
$$= 242cm^2$$
Similarly, area of yellow portion is:

$= \frac{1}{2} \times 968 cm^2 = 484 \text{cm}^2$

In triangle PCQ, Let PC = a = 20cm, CQ = b = 20cm, and PQ = c = 14cm.
Now, semi-perimeter of triangle PCQ will be:

$$s = \frac{a+b+c}{2}$$

$$= \frac{20+20+14}{2}$$

$$= \frac{54}{2}$$

$$= 27cm$$

So, area of triangle PCQ $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{27 \times (27-20) \times (27-20)(27-14)}$$

$$= \sqrt{27 \times 7 \times 7 \times 13}$$

$$= \sqrt{3 \times 3 \times 3 \times 7 \times 7 \times 13}$$

$$= 21\sqrt{39}$$

$$= 21 \times 6.24$$

$$= 131.04 \text{cm}^2$$

Since, the total area of green portion = $242 \text{ cm}^2 + 131.04 \text{ cm}^2 = 373.04 \text{ cm}^2$

Therefore, the paper required for each shade to make a kite is red paper = 242cm^2, yellow paper = 484cm^2, and green paper = 373.04cm^2.

2. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Solution:

Given: the perimeter of a triangle is 50 cm.

Now, semi-perimeter(s) of the triangle is $= \frac{\text{Perimeter of triangle}}{2} = \frac{50}{2} = 25$

Suppose that the smaller side of the triangle be a = x cm. So, the second side will be b= (x+4) cm and 3rd side will be c = (2x-6)cm.

Now, perimeter of triangle = a + b + c = x +(x+4)+(2x-6)
50 cm= (4x – 2) cm
50 =4x – 2
4x = 50 + 2
4x =52

$$x = \frac{52}{4}$$

$$x = 13$$

Since, the three side of the triangle are:
a = x = 13,
b= x+ 4 = 13 + 4 =17
c = 2x -6 = $2\times13-6=26-6=20$.

So, area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

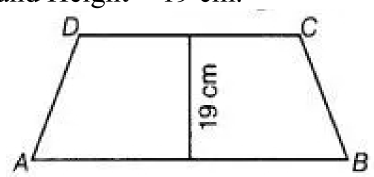
$$=\sqrt{25\times(25-13)\times(25-17)\times(25-20)}$$
$$=\sqrt{25\times12\times8\times5}$$
$$=\sqrt{5\times5\times4\times3\times4\times2\times5}$$
$$=5\times4\times20\sqrt{30}cm^2$$
$$=20\sqrt{30}cm^2$$

Therefore, the area of triangle is $20\sqrt{30}cm^2$.

3. The area of a trapezium is 475 cm² and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other.

Solution:

Given:
Area of a trapezium = $475cm^2$ and Height = 19 cm.



According to the question, let one sides of trapezium is x. So, another side will be x + 4.

Now, Area of trapezium = $\frac{1}{2}\times(\text{Sum of the parallel sides})\times\text{Height}$

$$475=\frac{1}{2}\times(x+x+4)\times19\text{cm}$$
$$2x+4=\frac{950}{19}$$
$$=50$$
$$2x=50-4$$
$$2x=46$$
$$x=23$$

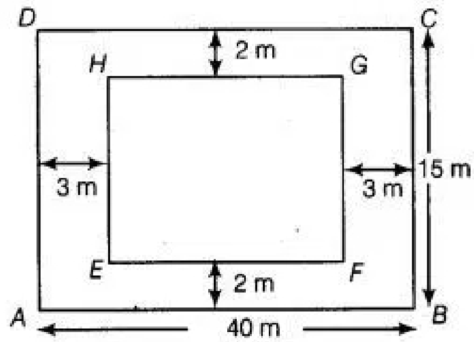
Therefore, the length of the parallel side of trapezium are x = 23 cm and x + 4 = 23 + 4 =27 cm.

4. A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a

minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of other sides. Find the largest area where house can be constructed.

Solution:

Given: Let a rectangular plot ABCD is constructing a house, having a measurement of 40 m long and 15 m in the front.



According to the question,

Length of inner-rectangle (EF) = 40 -3 -3 = 34m

And breadth of inner rectangle (FG) = 15 – 2 -2 =11m

Now, area of inner rectangle (EFGH) will be = Length x Breadth

$$= EF \times FG$$

$$= 34 \times 11 \text{m}^2$$

$$= 374 \text{m}^2$$

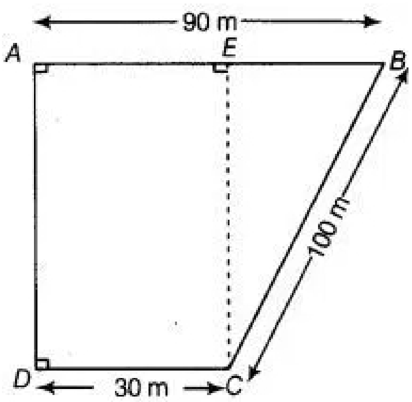
Therefore, the largest area where house can be constructed = 374m^2.

5. A field is in the shape of a trapezium having parallel sides 90 m and 30 m. These sides meet the third side at right angles. The length of the fourth side is 100 m. If it costs Rs 4 to plough 1m² of the field, find the total cost of ploughing the field.

Solution:

Given: In the trapezium ABCD, the two parallel sides are AB = 90 m, CD = 30 m, and $EC \perp AB$.

So, EB = AB – EA = 90 m - 30 m = 60m



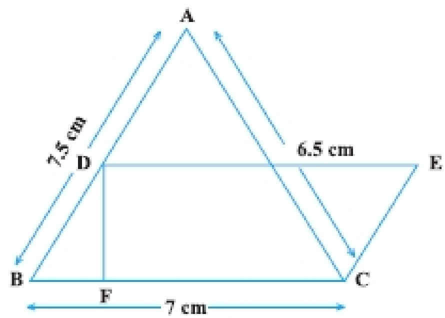
Now, in triangle BEC,

$$(BC)^2 = (BE)^2 + (EC)^2$$
$$100^2 = 60^2 + (EC)^2$$
$$(EC)^2 = 10000 - 3600$$
$$(EC)^2 = 6400$$
$$EC = \sqrt{6400}$$
$$EC = 80m$$

$$\textit{Now, area of trapezium ABCD} = \frac{1}{2} \times (\text{Sum of parallel sides}) \times (\text{Distance between parallel sides})$$
$$= \frac{1}{2} \times (AB + CD) \times EC$$
$$= \frac{1}{2} \times (90 + 30) \times 80$$
$$= \frac{1}{2} \times 120 \times 80$$
$$= 4800\text{m}^2$$

The cost of ploughing the field of 1m^2 is Rs. 4.
Now, The cost of ploughing the field of 4800m^2 area $= 4800 \times \text{Rs. } 4 = \text{Rs. } 19200$.
Therefore, the total cost of plughing the field is Rs. 19200.

6. In Fig., ΔABC has sides AB = 7.5 cm, AC = 6.5 cm and BC = 7 cm. On base BC a parallelogram DBCE of same area as that of ΔABC is constructed. Find the height DF of the parallelogram.



Solution:

Given: in triangle ABC, the sides are AB = a = 7.5 cm, BC = b = 7 cm, and CA = c = 6.5 cm.

Now, semi-perimeter of a triangle will be:

$$s = \frac{a+b+c}{2}$$
$$= \frac{7.5+7+6.5}{2}$$
$$= \frac{21}{2}$$
$$= 10.5$$

So, area of triangle ABC = $\sqrt{s(s-a)(s-b)(s-c)}$ [By heron's formula]

$$= \sqrt{10.5\times(10.5-7.5)(10.5-7)(10.5-6.5)}$$
$$= \sqrt{10.5\times3\times3.5\times4}$$
$$= \sqrt{441}$$
$$= 21\text{cm}^2$$

Also, the area of parallelogram BCED will be = Base $\times$Height

$$= BC\times DF$$
$$= 7\times DF$$

Now, according to the question,

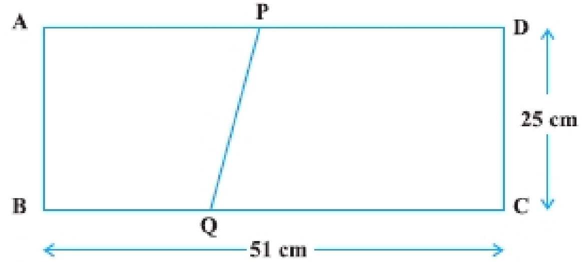
Area of triangle ABC = Area of parallelogram BCED

$$21 = 7\times DF$$
$$DF = \frac{21}{4}$$
$$DF = 3\text{cm}$$

Hence, the height of parallelogram BCED is 3 cm.

7. The dimensions of a rectangle ABCD are 51 cm × 25 cm. A trapezium PQCD with its parallel sides QC and PD in the ratio 9 : 8, is cut off from the

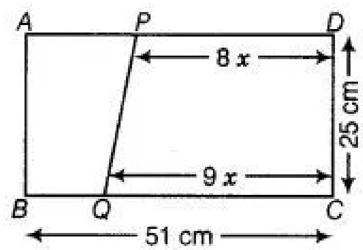
rectangle as shown in the Fig. If the area of the trapezium PQCD is $\frac{5}{6}$th part of the area of the rectangle, find the lengths QC and PD.



Solution:

Given: ABCD is a rectangle, where AB = 51 cm and BC = 25 cm.

The parallel sides QC and PD of the trapezium PQCD are in the ratio of 9 : 8. Let QC = 9x and PD = 8x.



Now, the area of trapezium PQCD:

$=\frac{1}{2}\times(\text{Sum of parallel sides})\times(\text{Distance between parallel sides})$

$=\frac{1}{2}\times(9x+8x)\times 25\text{cm}^2$

$=\frac{1}{2}\times 17x\times 25$

Again, area of rectangle ABCD = $BC\times CD=51\times 25$

Now, according to the question,

Area of trapezium PQCD = $\frac{5}{6}\times$ Area of rectangle ABCD

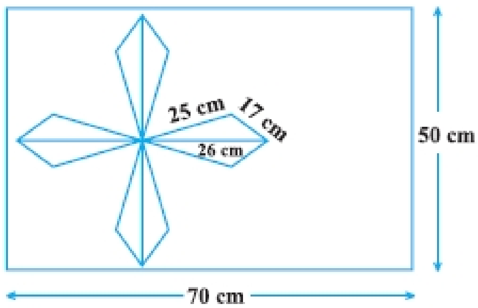
$$\frac{1}{2}\times 17x\times 25=\frac{5}{6}\times 51\times 25$$

$$x=\frac{5}{6}\times 51\times 25\times 2\times\frac{1}{17\times 25}$$

$$x=5$$

Therefore, the length of the trapezium PQCD, QC = 9x $=9\times 5=45cm$ and, PD = 8x = $8\times 5=40cm$.

8. A design is made on a rectangular tile of dimensions 50 cm × 70 cm as shown in Fig. The design shows 8 triangles, each of sides 26 cm, 17 cm and 25 cm. Find the total area of the design and the remaining area of the tile.



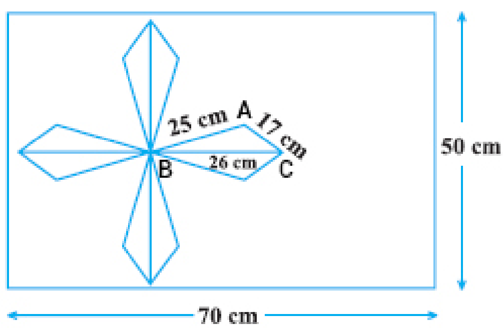
Solution:

Given: the dimension of the rectangular tile are 50 cm × 70cm.

So, area of the rectangular tile = 50 cm × 70 cm =$3500\,\text{cm}^2$.

See the given figure in the question, the sides of the triangle ABC be:

$a = 25cm, b = 17cm$, and $c = 26cm$



Since, semi-parameter(s) of triangle be:

$$s = \frac{a+b+c}{2}$$

$$= \frac{25+17+26}{2}$$

$$= \frac{68}{2}$$

$$= 34$$

So, area of triangle ABC = $\sqrt{s(s-a)(s-b)(s-c)}$ [By heron's formula]

Since, Total area of eight triangle = $8 \times$ Area of triangle ABC

$= 204 \times 8$

$= 1632\text{cm}^2$

The area of the design will be equal to the area of eight triangle that is 1632cm^2.

Now, remaining area of the tile = Area of the rectangle – Area of the design =
$3500\text{cm}^2 - 1632\text{cm}^2 = 1868\text{cm}^2$

Therefore, total area of the design is 1632cm^2 and the remaining area of the tile is 1868cm^2.